WAVE FORCE MODELING

INTRODUCTION

As a submarine operates near the free surface, it encounters complex forces which may cause unsatisfactory or unstable depth control. The lift and moment from incident waves increase in an exponential manner as the surface is approached. To maintain a desired depth, the ship's ballast is adjusted to counteract steady forces. Control surfaces are used to counter dynamic changes. A small depth excursion or change in forces can overwhelm the planes and cause a loss of depth control. The consequences range from losing radio reception to compromising the ship's mission.

The effects of incident waves on a submerged body can be divided up in several categories. The largest, the first order forces act at the incident wave frequency. These forces move the submarine, but usually result in oscillations about a mean state. Second order forces, which are the result of wave diffraction and wave interaction, have several different frequency components.

Wave diffraction of a single frequency wave results in a steady force and a varying force at twice the wave frequency. The double frequency force is typically neglected, as the large inertia of the submarine effectively filters it. Interactions of waves at different frequencies also results in forces. These consist of a component acting at the sum of the wave frequencies and a component acting at the difference of the wave frequencies. The sum frequency force is typically neglected, as it is also filtered by submarine's inertia. The difference frequency component results in a slowly varying force on the submarine.

The slowly varying forces are the principle cause of difficult periscope depth control (Ni, Zhang, and Dai, 1994). They are compensated for using control surfaces and occasional adjustments of trim.

During the design phase, engineering decisions are made which will determine the ship's ability to remain at periscope depth. Of these, the most critical are the height of the sail and control surface sizes. Every foot added to the sail gives a deeper periscope depth. Larger planes improve the operator's ability to compensate for changes in suction forces. However, these improvements are not without cost. The sail and other appendages are a large fraction of the total drag, and can restrict the ship's top speed. Larger movable control surfaces can adversely affect the high speed casualty recoverability (Jackson, 1992, p. 15-9).

The goal of this thesis is not to provide new tools for the designer, but rather new means to enhance control for the operators of current submarines. Due to this focus, simplified means of modeling the wave forces for a few specific cases will be used.

REVIEW OF LINEAR DEEP WATER WAVES

The pertinent features of linear deep water waves will be reviewed to provide background for the following sections. The coordinate system used for the examples is shown in Figure 5. For the examples in this section, it will be assumed that the submarine is oriented with the bow pointing into the page. Consistent with the global coordinate system from Chapter II, the distance from the surface to the submarine centerline is z. The submarine diameter is D.

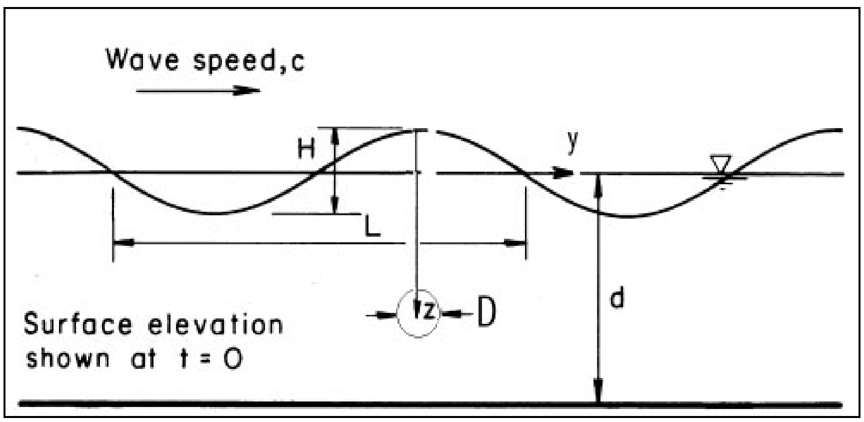

Figure 5. Coordinate Definition for plane progressive wave, adapted from Sarpkaya and Isaacson (1981, p. 151)

For a wave of wavelength L, a wave number, *k*, can be defined.

$$k = \frac{2\pi}{L} \tag{30}$$

Assuming that fluid is incompressible and inviscid Laplace's equation can be applied. It is thus desired to find a solution to:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \tag{31}$$

To this, the boundary conditions at the free surface, and the bottom must be applied:

$$\frac{\partial \phi}{\partial z} = 0 \text{ , at } z = d \text{ (no flow through ocean bottom)} \qquad (32)$$

$$\frac{\partial \eta}{\partial t} + \frac{\partial \phi}{\partial x}\frac{\partial \eta}{\partial x} + \frac{\partial \phi}{\partial z} = 0 \text{ , at } z = \eta \text{ (zero velocity normal to ocean surface)} \qquad (33)$$

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}\left[\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial z}\right)^2\right] + g\eta = f(t) \text{ , at } z = \eta \qquad (34)$$

For small amplitude waves in deep water, the following solution can be obtained (adapted from Sarpkaya and Isaacson, 1981, p. 159):

$$\phi = \frac{\pi H}{kT} e^{-kz} \sin(\omega t) \qquad (35)$$

$$c^2 = \frac{\omega^2}{k^2} = \frac{g}{k} \qquad (36)$$

$$L = \frac{gT^2}{2\pi} \qquad (37)$$

$$\eta = \frac{H}{2}\cos(\omega t) \qquad (38)$$

$$\zeta = -\frac{H}{2} e^{-kz} \cos(\omega t) \qquad (39)$$

$$\dot{\zeta} = \omega \frac{H}{2} e^{-kz} \sin(\omega t) \qquad (40)$$

$$\ddot{\zeta} = \omega^2 \frac{H}{2} e^{-kz} \cos(\omega t) \qquad (41)$$

$$\xi = -\frac{H}{2} e^{-kz} \sin(\omega t) \qquad (42)$$

$$\dot{\xi} = -\omega \frac{H}{2} e^{-kz} \cos(\omega t) \qquad (43)$$

$$\ddot{\xi} = \omega^2 \frac{H}{2} e^{-kz} \sin(\omega t) \qquad (44)$$

where η is the distance from the surface to the average level ($z = 0$), ω is the angular frequency of the incident wave, ζ is the displacement of a particle in the x direction, and ξ is the displacement of a particle in the z direction.

A key parameter in oscillating flows is the Keulegan-Carpenter number:

$$K = \frac{U_{mean} T}{D} \tag{45}$$

where U_{mean} is the average velocity across the characteristic dimension D.

By taking the average of the velocity given in Equation (40) , and substitution into Equation (45), the expression for the Keulegan-Carpenter can be reduced to the following:

$$K = \frac{2H}{D} e^{-kz} \tag{46}$$

Equation (46) is the Keulegan-Carpenter number based on the cross flow velocity of the undisturbed wave at the same depth as the centerline of the submarine hull.

WAVE FORCE REGIMES

There are different regimes of interaction between a submerged body and a wave field. Broadly, they can be broken into several areas. Inertial interaction, where the body acts like a particle in the wave field. Wave diffraction, where the bodies influence upon the wave field is accounted for. Finally, there are flow separation (viscous) effects. The relative importance of each of these effects can be determined by examining the relationship the body size to the wave parameters. (Sarpkaya and Isaacson, 1981, pp. 381-386)

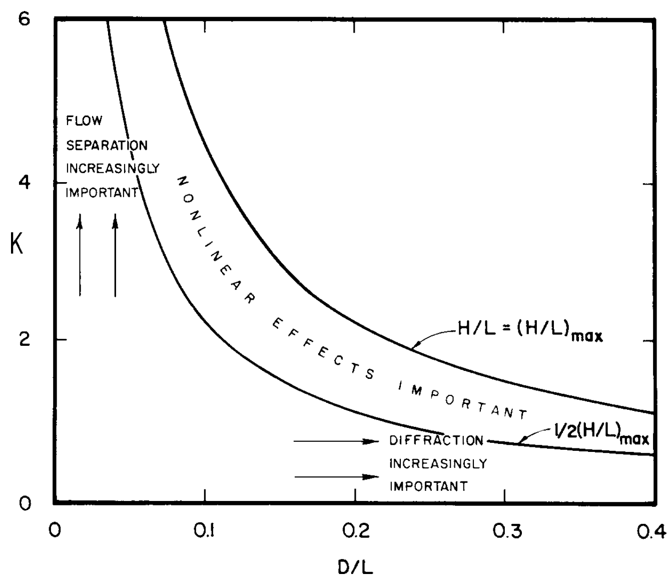

Figure 6. Wave force regimes (Sarpkaya and Isaacson, 1981, pg. 385)

To estimate the significant effects for a typical SSN, a typical operating condition is assumed. For a 300 foot submarine with a 35 foot diameter, a typical periscope operating depth would be about 50 feet from the centerline of the ship to the free surface. Using average values for sea states three and four and assuming deep water compared to the wavelength, the following quantities were calculated at a depth of 50 feet:

Parameter	Sea State 3	Sea State 4
Significant Wave Height	3	6
Average Period	6.623501	7.154522
Wave Length	224.6467	262.1114
Wave Number	0.027969	0.023971
K	0.042339	0.103414
D/L	0.1558	0.133531

Table 2. Estimated Wave Loading Parameters

The Diameter/Wavelength (D/L) ratios and the Keulegan-Carpenter numbers of Table 2 show that for the sea states of interest, wave diffraction much more significant than viscous forces. It can be concluded that an inviscid analysis should give good results for the wave forces. However, this is only rigorous for an unappended hull, as the control surfaces and sail on an actual submarine will experience viscous effects.

SOLUTION FROM SLENDER BODY THEORY

Wave force solutions for several specific cases were generated for the SUBOFF by the SSBN Security Department of the Johns Hopkins University Applied Physics Laboratory. A slender body solution with some three dimensional corrections was used. The specific method used for the generation of the first order motions and second order forces is detailed by O'Dea and Barr (1976, pp. 7-25).

A seaway approximation consisting of a small number of regular waves was used to model sea states three and four. For each sea state, the resulting data were separated into two categories. The effects of the first order forces were given in terms of body motions. The effects of the steady second order forces and the difference interaction forces were provided in pounds force.

Seaway model

A random seaway can be represented by the superposition of a large number of regular waves. The seaway was approximated by superimposing n regular waves. The frequency and height of these waves was determined using the Bretschneider spectrum. It gives the spectral density in terms of the significant wave height, H_s, and the peak frequency, ω_o.

$$S(\omega) = \frac{5H_s^2}{16\omega_o(\omega/\omega_o)^5} e^{\left[-\frac{5}{4}\left(\frac{\omega}{\omega_o}\right)^{-4}\right]} \tag{47}$$

To model sea state three, a significant wave height of three feet was used, with a central frequency of 0.836 radians per second. This results in the following spectrum:

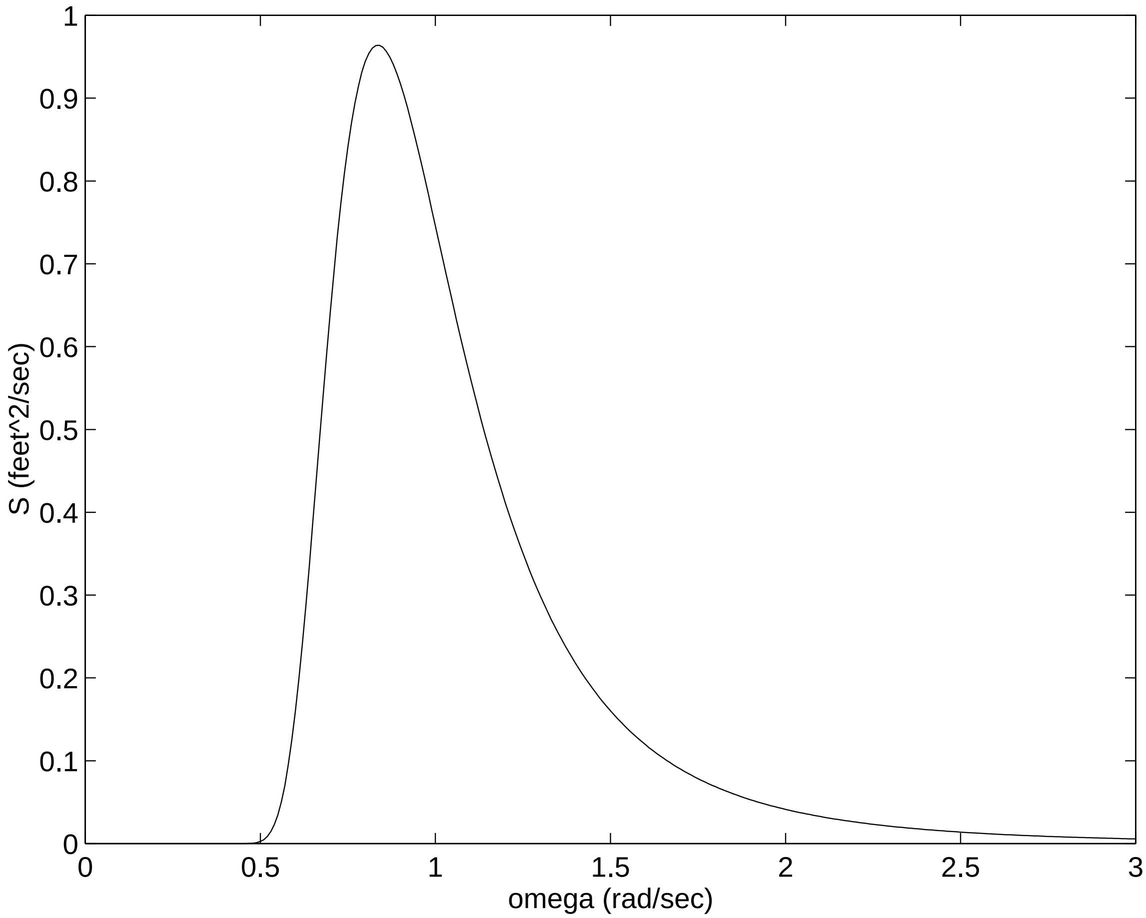

Figure 7. Example Sea State three spectrum

Figure 7 gives a statistical picture of the seaway, but is not immediately useful for time domain simulation. One way to obtain a time history is to represent this stationary process as a the sum of a series of sine waves:

$$\eta(t) = \sum_{i=1}^{n} A_i \sin(\omega_i t + \alpha_i) \tag{48}$$

Where A_i is the amplitude of the i^{th} wave , and α_i is its randomly chosen phase angle.

If the number of sine waves is reasonable large, and the frequencies and amplitudes of each component are chosen to achieve the same energy as the section of spectrum it represents, Equation (48) will give a good representation of the ocean surface.

The method chosen was to divide the spectra into n segments of equal areas. This results in n sine waves all with equal amplitudes. Integration of Equation (48) yields:

$$\int_0^\infty S(\omega)d\omega = \frac{H^2{}_s}{16} \tag{49}$$

Because the spectrum extends to infinity, it was chosen truncate the spectrum at a point where the area was a fraction C of the total area. The amount of area to be represented by each sine wave is equal to its mean square value. So the amplitude of each sine wave is equal to the square root of the area it represents times the square root of two.

$$A_i = \frac{H_s}{2}\sqrt{\frac{C}{2n}} \tag{50}$$

Equation (48) can be integrated up to some frequency $\hat{\omega}_i$, which represents the frequency at which the spectral area is equal to iC/n times the total area.

$$\int_0^{\hat{\omega}_i} S(\omega)d\omega = \frac{H^2{}_s}{16}\frac{i}{n}C \tag{51}$$

Solving Equation (51) for $\hat{\omega}_i$ yields:

$$\hat{\omega}_i = \omega_o\left[\frac{4}{5}\ln\left(\frac{Ci}{N}\right)\right]^{\frac{1}{4}} \tag{52}$$

Because the spectral level is insignificant below ω equal to $0.6\omega_o$, the frequency if the first segment was determined as follows:

$$\omega_1 = \frac{(0.6\omega_o + \hat{\omega}_1)}{2} \tag{53}$$

The remainder of the frequencies were determined by taking the midpoint of the frequencies at either side of the area segment.

$$\omega_i = \frac{\hat{\omega}_{i-1} + \omega_i}{2}\text{, for } i = 2 \text{ to } n \tag{54}$$

Figure 8 illustrates the method used, approximating the spectrum with sinusoids. Nineteen equal area sections are divided, with the center frequency of each segment marked with a circle.

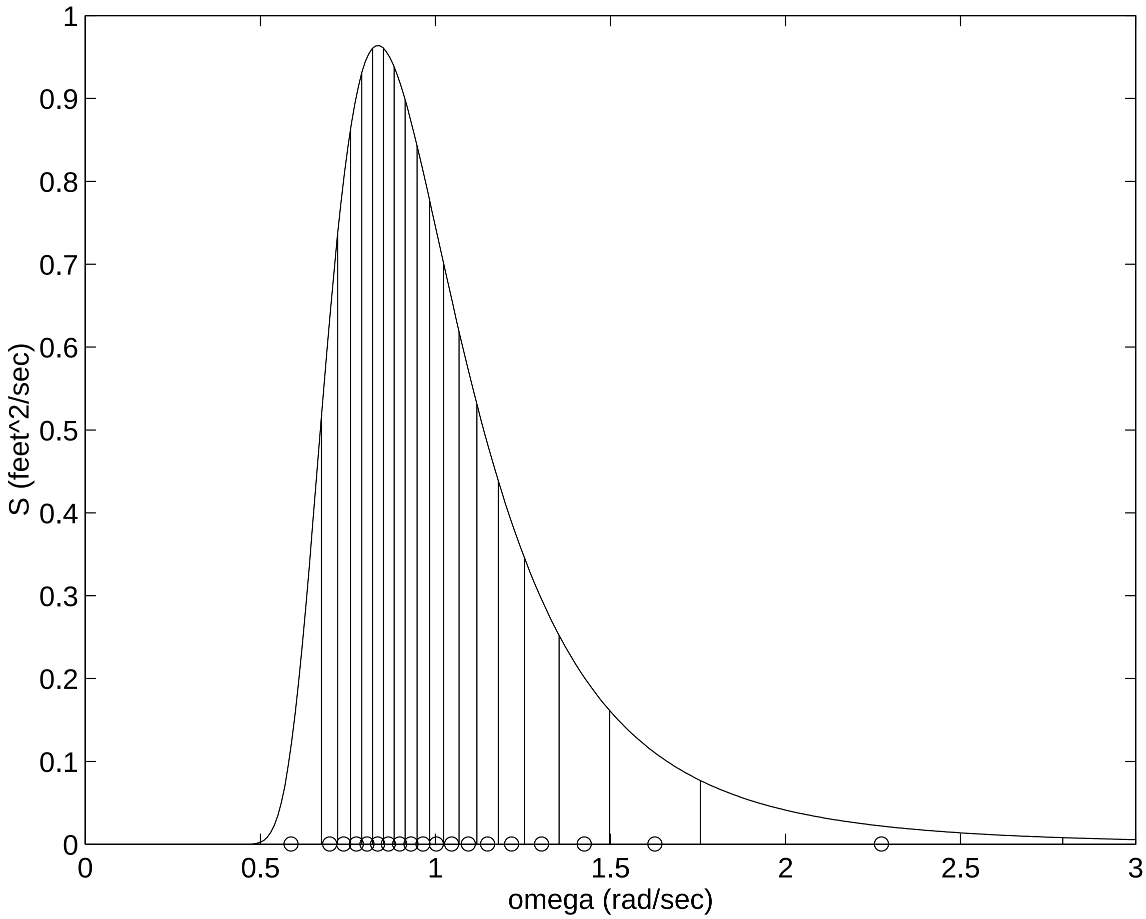

Figure 8. Spectra area division and mean frequencies

Figure 9 shows the ocean surface which results from the use of this method for the case of sea state three, peak frequency of 0.862 radians per second. Nineteen sinusoids were used to approximate the spectra, and the phase angles were randomly chosen.

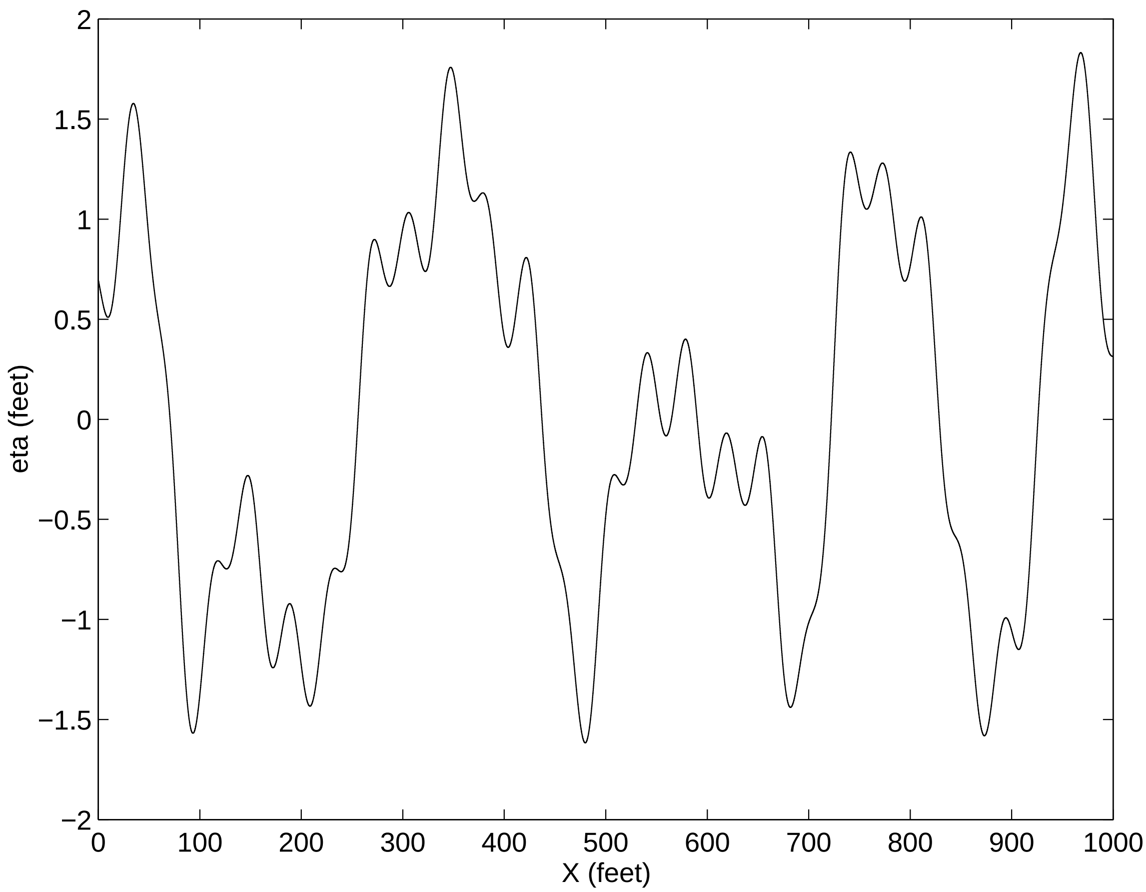

Figure 9. Sea surface approximation for sea state three using nineteen sinusoids

First order forces

The first order wave effects were provided in the form of submarine motions. They were given as a series of phasors, the real part of the summation representing the actual perturbation caused by the first order wave forces.

$$z(t) = \sum_{i=1}^{n} Z_i e^{-i(\omega_i t+\alpha_i)} \tag{55}$$

$$\theta(t) = \sum_{i=1}^{n} \theta_i e^{-i(\omega_i t+\alpha_i)} \tag{56}$$

Because the first order motions were provided for a specific depth, it was required to correct Equations (55) and (56) for depth. The first order motions roughly correspond to the particle motions given by Equation (42), so an exponential decay was used to derive the following correction factor:

$$\frac{e^{-k_i z}}{e^{-k_i z_o}} \tag{57}$$

Application of Equation (57) to Equations (55) and (56) results in:

$$z(t) = \sum_{i=1}^{n} Z_i e^{-\sqrt{-1}(\omega_i t+\alpha_i)-k_i(z-z_o)} \tag{58}$$

$$\theta(t) = \sum_{i=1}^{n} \theta_i e^{-\sqrt{-1}(\omega_i t+\alpha_i)-k_i(z-z_o)} \tag{59}$$

The displacements given by Equations (58) and (59) are not suitable for inclusion in the submarine equations of motion. For this, an acceleration is required. Differentiating twice with respect to time results in:

$$\ddot{z}(t) = -\sum_{i=1}^{n} \omega_i{}^2 Z_i e^{-i(\omega_i t+\alpha_i)-k_i(z-z_o)} \tag{60}$$

$$\ddot{\theta}(t) = -\sum_{i=1}^{n} \omega_i{}^2 \theta_i e^{-i(\omega_i t+\alpha_i)-k_i(z-z_o)} \tag{61}$$

Equations (60) and (61) were incorporated as force and moment disturbances in the equations of motion found in Chapter II. To test the validity of this approach, an open loop simulation was performed using the accelerations from Equations (60) and (61) for one sea state and heading. Figure 10 shows the results of this simulation, as well as the expected first order motions. The upper curve shows the expected first order motions, and the lower curve shows the results of integrating Equations (7) through (11) with the accelerations from Equations (60) and (61). Although there was some drifting motion, the character of motion and the approximate amplitude of each cycle of motion very close. The drifting motion was a result lack of the lack of open loop depth stability, which is characteristic of submarines.

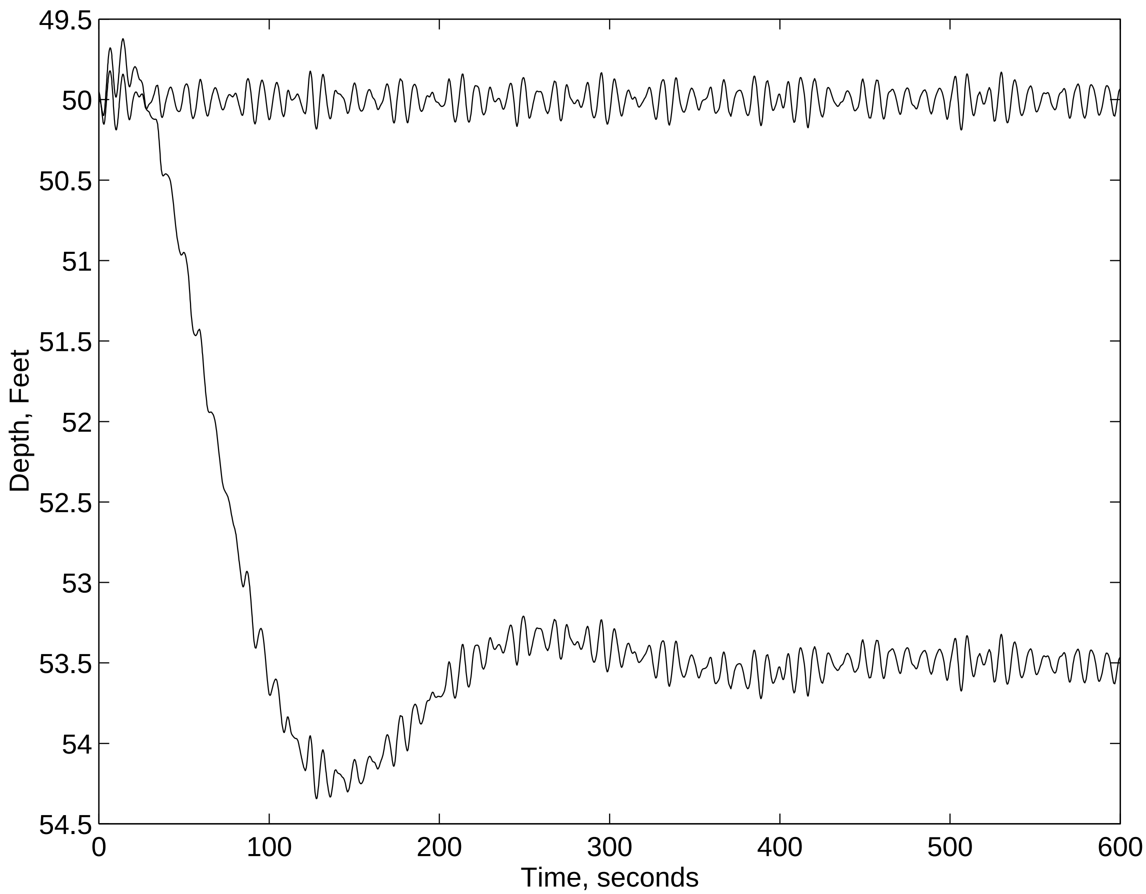

Figure 10. Submarine response to first order accelerations, and expected response

Second order forces

For a particular depth and wave time history, the second order forces were given in the following form:

$$Z(t) = \sum_{i=1}^{n} \sum_{j=1}^{n} F_{ij} e^{\left(i((|\omega_i - \omega_j|)t + \alpha_i + \alpha_j)\right)} \tag{62}$$

$$M(t) = \sum_{i=1}^{n} \sum_{j=1}^{n} M_{ij} e^{\left(i((|\omega_i - \omega_j|)t + \alpha_i + \alpha_j)\right)} \tag{63}$$

$Z(t)$ represents the force acting at the body fixed coordinate system in the z direction and $M(t)$ is the moment acting about the y axis. It should be noted that Equations (63) and (65) include the slowly varying forces ($i \neq j$) and the steady forces ($i = j$).

It can be determined from analysis of the changes of second order forces with respect to depth given by Crook (1994, pp. 61,62) that the steady forces with the following exponential decay factor:

$$e^{-2kz} \tag{64}$$

The slowly varying order wave forces vary with depth according to the sum of the wave numbers:

$$\frac{e^{-(k_i+k_j)z}}{e^{-(k_i+k_j)z_o}} \tag{65}$$

Application of Equations (64) and (65) to Equations (62) and (63) results in:

$$Z(t) = \sum_{i=1}^{n} \sum_{j=1}^{n} F_{ij} e^{\left(i((\left|\omega_i-\omega_j\right|)t+\alpha_i+\alpha_j)-(k_i+k_j)(z-z_o)\right)} \tag{66}$$

$$M(t) = \sum_{i=1}^{n} \sum_{j=1}^{n} M_{ij} e^{\left(i((\left|\omega_i-\omega_j\right|)t+\alpha_i+\alpha_j)-(k_i+k_j)(z-z_o)\right)} \tag{67}$$

The real portion of Equations (66) and (67) represents the steady and slowly varying second order wave forces acting on the submarine, with correction for depth.

Inclusion of wave forces in equations of motion

The first order accelerations and second order forces had to be combined to form the force and moment disturbance accelerations for use in the deeply submerged equations of motion (Equations (7) and (8)).

$$\begin{bmatrix} F_d(t) \\ M_d(t) \end{bmatrix}_{\substack{Incident \\ Waves}} = \begin{bmatrix} \ddot{z}(t) \\ \ddot{\theta}(t) \end{bmatrix} + \overline{M}^{-1} \begin{bmatrix} Z(t) \\ M(t) \end{bmatrix} \tag{68}$$

CONCLUDING REMARKS

An elementary review of linear wave theory was presented. The case of interest, a submarine at periscope depth, was examined to determine the salient elements of its interaction with the incident waves. The parameters suggested that the major features of the incident wave effects on the submarine could be determined by using a potential analysis with inertial and diffraction forces accounted for.

The Bretschneider spectrum was used to determine the spectral density functions of the sea states of interest. For the purpose of time domain simulation, the spectrum was approximated by the superposition of a number of regular waves with randomly chosen phase angles.

The first order force transfer function and second order forces response amplitude operators were provided for the SUBOFF for a nominal speed and depth. Approximate depth scaling was introduced to allow use at depths other than nominal.